

INdAM Workshop 2025

Differential Equations and Nonlinear Models - DENoM

INdAM, Rome - June 9-13, 2025

Plenary Speakers

Nicolas Bacaër (Institut de Recherche pour le Développement, Paris)

Andrea Corli (Università degli Studi di Ferrara)

Benedetto Piccoli (Rutgers University – Camden)

Carlota Rebelo (Universidade de Lisboa)

Invited Speakers

Irene Benedetti (Università degli Studi di Perugia)

Pierluigi Benevieri (Universidade de São Paulo)

Timoteo Carletti (Université de Namur)

Rossella Della Marca (Università degli Studi di Napoli Federico II)

Maria Laura Delle Monache (University of California, Berkeley)

Paolo Gidoni (Università degli Studi di Udine)

Roberto Livrea (Università degli Studi di Palermo)

Nastassia Pouradier Duteil (INRIA Paris)

Cinzia Soresina (Università degli Studi di Trento)

Sergey Tikhomirov (PUC-Rio)

Wahid Ullah (Università degli Studi di Trieste)

Poster Presenters

Giulia Duricchi (Università degli Studi di Modena e Reggio Emilia)

Ludmila Linhartová (Masarykova Univerzita Brno)

Natnael Gezahegn Mamo (Università degli Studi di Trieste)

Emanuele Pastorino (Politecnico di Milano)

Schedule of the Workshop

	Monday, June 9
15:00-15:30	<i>Welcome</i>
15:30-16:45	C. REBELO - <u>Mini-course</u> : Population dynamics models with seasonality
16:50-17:15	<i>Coffee break</i>
17:15-18:30	N. BACAËR - <u>Mini-course</u> : Epidemic models and ordinary differential equations
	Tuesday, June 10
08:45-09:35	I. BENEDETTI - Nonlocal differential problems in abstract spaces
09:40-10:55	N. BACAËR - <u>Mini-course</u> : Epidemic models and ordinary differential equations
11:00-11:25	<i>Coffee break</i>
11:25-12:40	C. REBELO - <u>Mini-course</u> : Population dynamics models with seasonality
12:45-13:35	P. GIDONI - Limit cycle and asymptotic gait for a dynamic model of rectilinear locomotion
13:40-15:00	<i>Lunch</i>
15:00-15:50	C. SORESINA - Derivation of cross-diffusion models in population dynamics: dichotomy, time-scales, and fast-reaction
15:55-16:45	S. TIKHOMIROV - Travelling waves in tubes model of gravitational fingering
	Wednesday, June 11
08:45-09:35	R. DELLA MARCA - On the optimal control of epidemic models
09:40-10:30	<u>Poster launches</u> : G. DURICCHI, L. LINHARTOVÁ, N.G. MAMO, E. PASTORINO
10:35-11:00	<i>Coffee break</i>
11:00-11:50	R. LIVREA - Nonlinear differential problems via variational, set-valued and topological methods
11:55-12:45	W. ULLAH - Multiplicity results for boundary value problems associated with Hamiltonian systems
12:50-15:00	<i>Lunch</i>
15:00-16:45	<i>Group work and discussions</i>
20:00	Social dinner at <i>Ristorante I Fratelli</i>
	Thursday, June 12
08:45-09:35	M.L. DELLE MONACHE - Coupled PDE-ODE models and control strategies for mixed autonomy traffic flow
09:40-10:55	A. CORLI - <u>Mini-course</u> : Traveling waves for parabolic equations with degenerate diffusivities
11:00-11:25	<i>Coffee break</i>
11:25-12:40	B. PICCOLI - <u>Mini-course</u> : Control of multi-agents systems
12:45-13:35	P. BENEVIERI - Bifurcation results for a delay differential system
13:35-15:00	<i>Lunch</i>
15:00-15:50	T. CARLETTI - Global synchronization on networks and beyond
15:55-16:45	N. POURADIER DUTEIL - Mean-field limit of particle systems over hypergraphs
	Friday, June 13
09:15-10:30	B. PICCOLI - <u>Mini-course</u> : Control of multi-agents systems
10:35-11:00	<i>Coffee break</i>
11:00-12:15	A. CORLI - <u>Mini-course</u> : Traveling waves for parabolic equations with degenerate diffusivities
12:20-12:45	<i>Closing</i>

Mini-Courses

Epidemic models and ordinary differential equations

Nicolas BACAËR

Institut de Recherche pour le Développement, Paris, France

In these two lectures we will focus on epidemic models, which are systems of ordinary differential equations. The main focus will be on the SIR and SEIR models, including the not so well known problem of estimating the time of the epidemic peak. The basic reproduction number is related to the linearized system near the disease-free steady state. We will also consider the case of periodic coefficients and of stochastic models that can be studied with ordinary differential equations.

References

- [1] Matematica ed epidemie, <https://hal.science/hal-03885380>
- [2] Una breve storia della dinamica matematica delle popolazioni, <https://hal.science/hal-03313544>

Traveling waves for parabolic equations with degenerate diffusivities

Andrea CORLI

Università degli Studi di Ferrara, Italy

The mini-course focuses on traveling-wave solutions for degenerate parabolic equations, in one spatial dimension. By degeneracy we mean that diffusivity can vanish at some points or even be negative in some intervals. Several motivations drawn from applied mathematics are given. Addressing a broad audience, the first part of the course briefly recalls the classical traveling-wave theory for strictly positive diffusivities, with and without a source term. The second part covers a review of recent results, together with an illustration of demonstration methods. Applications to biomathematics and traffic flows are also shown.

Control of multi-agents systems

Benedetto PICCOLI

Rutgers University – Camden, USA

In this mini-course we will present challenges and opportunities in control of large systems of agents. Applications to vehicular traffic, socio-dynamic systems, and pedestrian safety will be illustrated.

Population dynamics models with seasonality

Carlota REBELO
Universidade de Lisboa, Portugal

In these two lectures we will focus on population dynamics models with seasonality. We will recall the definition of offspring number and give results about extinction, persistence and coexistence depending on whether this number is below or above one. Finally results on the stability of the periodic orbits will be mentioned.

References

- [1] I. Coelho, C. Rebelo and E. Sovrano, Extinction or coexistence in periodic Kolmogorov systems of competitive type, *Discrete Contin. Dyn. Syst.* 41 (2021), 5743–5764
- [2] M. Garrione and C. Rebelo, Persistence in seasonally varying predator-prey systems via the basic reproduction number, *Nonlinear Anal. Real World Appl.* 30 (2016), 73–98
- [3] V. Ortega and C. Rebelo, A note on stability criteria in the periodic Lotka–Volterra predator-prey model, *Appl. Math. Lett.* 145 (2023), article 108739
- [4] C. Rebelo, A. Margheri and N. Bacaër, Persistence in seasonally forced epidemiological models, *J. Math. Biol.* 64 (2012), 933–949
- [5] C. Rebelo and C. Soresina, Coexistence in seasonally varying predator-prey systems with Allee effect, *Nonlinear Anal. Real World Appl.* 55 (2020), 103–140

Invited Talks

Nonlocal differential problems in abstract spaces

Irene BENEDETTI, Università degli Studi di Perugia, Italy

This talk presents results on the existence and localization of solutions for nonlocal differential problems in abstract spaces. The differential equations under consideration involve a term governed by an m -dissipative maximal monotone operator, which may also be nonlinear. The proposed approach is based on fixed point theorems combined with so-called transversality conditions, offering a unifying framework for the study of diffusion models in various settings. This method covers both periodic and more general nonlocal initial conditions - such as multipoint or integral-type conditions - and can handle nonlinearities with superlinear growth, including cubic-type terms or nonlinearities depending on the integral of the solution, thus capturing behaviors characteristic of nonlocal diffusion phenomena.

The talk is mainly based on the papers [1, 2, 3, 4].

References

- [1] I. Benedetti and S. Ciani, Evolution equations with nonlocal initial conditions and superlinear growth, *J. Differential Equations* 318 (2022), 270–297
- [2] I. Benedetti, N.V. Loi and V. Taddei, Nonlocal diffusion second order partial differential equations, *Discrete Contin. Dyn. Syst.* 37 (2017), 2977–2998
- [3] I. Benedetti, L. Malaguti and M.D.P. Monteiro Marques, Differential equations with maximal monotone operators, *J. Math. Anal. Appl.* 539 (2024), article 128484
- [4] I. Benedetti, L. Malaguti and V. Taddei, Nonlocal solutions of parabolic equations with strongly elliptic differential operators, *J. Math. Anal. Appl.* 473 (2019), 421–443

Bifurcation results for a delay differential system

Pierluigi BENEVIERI, Universidade de São Paulo, Brasil

This talk summarizes a recent paper [1] in which we present a global bifurcation result for periodic solutions of the following delayed first order system, depending on a real parameter $\lambda \geq 0$,

$$\begin{cases} s'(t) = Ds^0(t) - Ds(t) - \frac{\lambda}{\gamma} \mu(s(t))x(t) & t \geq 0 \\ x'(t) = x(t)[\lambda\mu(s(t-\tau)) - D] & t \geq 0, \end{cases} \quad (1)$$

in which the following conditions hold:

- (a) $s^0 : \mathbb{R} \rightarrow \mathbb{R}$ is continuous, positive and ω -periodic, where $\omega > 0$ is given,
- (b) $\mu : [0, +\infty) \rightarrow [0, +\infty)$ is C^2 and verifies $\mu(0) = 0$ and $\mu'(s) > 0$, for any $s \in [0, +\infty)$,
- (c) D, γ and the delay τ are positive constants.

System (1) has been studied in [2] and it represents a chemostat model, with a delay. The chemostat is a continuous bioreactor with a constant volume, in which one or more microbial species are cultivated in a liquid medium containing a set of resources with, in particular, a specific nutrient. The maps $s(t)$ and $x(t)$ are, respectively, the densities of the nutrient and of the microbial species at time t . The device receives continuously an input of liquid volume, described by $s^0(t)$, containing a variable concentration of the specific nutrient. It expulses continuously towards the exterior an output of liquid volume containing a mixing of microbial biomass and nutrient. The model described by the system (1) assumes that the consumption of the nutrient has no immediate effects on the microbial growth, but we have a time interval $[t - \tau, t]$ in which the microbial species metabolize(s) the nutrient.

If (s, x) is any solution of (1) such that x vanishes at some t_0 , then x turns out to be identically zero. Thus, the first equation in system (1) becomes linear and has a unique ω -periodic solution, which is positive and can be written as

$$v^*(t) = \int_{-\infty}^t e^{-D(t-r)} Ds^0(r) dr.$$

For a sake of simplicity, assume that $\frac{1}{\omega} \int_0^\omega \mu(v^*(t)) dt = D$. In [2], the authors prove that

- (a) if $\lambda < 1$ (resp. $\lambda > 1$) and (s, x) is an ω -periodic solution, different from $(v^*, 0)$, then, $x(t) < 0$ (resp. $x(t) > 0$) for all $t \in \mathbb{R}$;
- (b) if $\lambda = 1$, no ω -periodic solution is different from $(v^*, 0)$.

Hence, it is quite natural to ask if $(v^*, 0)$ is a bifurcation point for ω -periodic solutions of (1) as well as to investigate the global behaviour of the bifurcating branches of such solutions. Here, we call ω -triple an element (λ, s, x) in which (s, x) is an ω -periodic solution of (1) corresponding to λ . Denote by E the Banach space $E := \mathbb{R} \times C_\omega^1 \times C_\omega^1$, where $C_\omega^1 = \{u \in C^1([0, \omega], \mathbb{R}) : u(0) = u(\omega) \text{ and } u'(0) = u'(\omega)\}$.

Our main result is the following:

There exist in E exactly two connected components $\mathcal{C}_+$ and $\mathcal{C}_-$ of nontrivial ω -triples, which are unbounded, contain $(1, v^, 0)$ in their closure and are such that every $(\lambda, s, x) \in \mathcal{C}_+$ verifies $\lambda > 1$, $0 < s < v^*$ and $x > 0$, while every $(\lambda, s, x) \in \mathcal{C}_-$ verifies $\lambda < 1$, $s > v^*$ and $x < 0$.*

The proof uses, among other tools, the Crandall-Rabinowitz local bifurcation theorem [4] and a concept of degree introduced in [3] for Fredholm maps of index zero between Banach spaces.

Based on a joint work with Pablo Amster (Universidad de Buenos Aires).

References

- [1] P. Amster and P. Benevieri, Global bifurcation results for a delay differential system representing a chemostat model, J. Differential Equations 434 (2025), article 113222
- [2] P. Amster, G. Robledo and D. Sepúlveda, Dynamics of a chemostat with periodic nutrient supply and delay in the growth, Nonlinearity 33 (2020), 5839–5860
- [3] P. Benevieri and M. Furi, A simple notion of orientability for Fredholm maps of index zero between Banach manifolds and degree theory, Ann. Sci. Math. Québec 22 (1998), 131–148
- [4] M.G. Crandall and P.H. Rabinowitz, Bifurcation from simple eigenvalues, J. Funct. Anal. 8 (1971), 321–340

Global synchronization on networks and beyond

Timoteo CARLETTI, Université de Namur, Belgium

Synchronization is the phenomenon according to which coupled nonlinear oscillators exhibit an unison rhythm and behave as a single oscillator without the need of any external control device, it is an emergent property of the system largely studied with many relevant applications. We present some results about global synchronization for identical coupled oscillators via networks or simplicial complexes. Then we will consider topological signals, i.e., dynamical variables defined on nodes, links, triangles, etc. of higher-order networks. The latter are attracting increasing attention from scholars but the investigation of their collective phenomena is only at its infancy. We combine topology and nonlinear dynamics to determine the conditions for global synchronization of topological signals on simplicial complexes [T. Carletti, L. Giambagli and G. Bianconi, Phys. Rev. Letters 130 (2023), article 187401]. We show that topological obstruction impedes odd dimensional signals to globally synchronize. We provide results where the coupling can be realized via the Laplace matrix or the Dirac one [T. Carletti, L. Giambagli, R. Muolo and G. Bianconi, J. of Physics: Complexity (2025), in press, arXiv:2410.15338 (2024)], allowing thus to couple signals of different dimension.

Acknowledgment. The presented work is the result of several projects realized with several colleagues, among which Prof. Ginestra Bianconi, Lorenzo Giambagli and Riccardo Muolo.

On the optimal control of epidemic models

Rossella DELLA MARCA, Università degli Studi di Napoli Federico II, Italy

Optimal control (OC) theory applied to epidemic models provided a useful framework to investigate issues concerning the amount of resources needed - e.g. for vaccination, drug treatment or isolation interventions - to effectively tackle epidemic events [4]. In the last decades, and to a larger extent after the beginning of the SARS-CoV-2 pandemic, the number of scientific works on the application of optimal control to infectious diseases has substantially increased, becoming more than 200 per year. In the context of such a dramatic increase in the usage of this class of models for epidemiological purposes, it is crucial to understand the potential and the limitations of such a tool. For this reason, we studied how relaxing both classical epidemiological assumptions and popular classes of control constraints affects the outcome of OC problems. We considered the OC of basic Susceptible–Infected–Recovered (SIR) models in the following relevant but unexplored frameworks: i) the case that the infectious period is Erlang–distributed, implying that the chance for an infected individual to recover depends on the time since infection, as it has been documented for a wide class of infectious diseases [2, 3]; ii) the case that the costs of the epidemics are related not only to epidemic size, but also to epidemic duration. Indeed, the minimization of outbreaks duration is a priority when the imposed sanitary restrictions involve travel bans (in human diseases) and export bans (in livestock diseases) [1].

References

- [1] L. Bolzoni, E. Bonacini, R. Della Marca and M. Groppi, Optimal control of epidemic size and duration with limited resources, *Math. Biosci.* 315 (2019), article 108232
- [2] L. Bolzoni and R. Della Marca, On the optimal vaccination control of SIR model with Erlang–distributed infectious period, *J. Optim. Theory Appl.* 205 (2025), article 39
- [3] L. Bolzoni, R. Della Marca and M. Groppi, On the optimal control of SIR model with Erlang–distributed infectious period: isolation strategies, *J. Math. Biol.* 83 (2021), article 36
- [4] O. Sharomi and T. Malik, Optimal control in epidemiology, *Ann. Oper. Res.* 251 (2017), 55–71

Coupled PDE-ODE models and control strategies for mixed autonomy traffic flow

Maria Laura DELLE MONACHE, University of California, Berkeley, USA

As autonomous vehicles (AVs) with features such as adaptive cruise control become more prevalent, understanding and controlling their interaction with human-driven traffic is critical. In this talk, we present a mathematical framework to analyze and control mixed traffic systems using a class of coupled PDE-ODE models. The macroscopic traffic flow is described by a scalar conservation law, while the dynamics of AVs are governed by systems of ordinary differential equations. These ODEs account for individual vehicle behaviors and may include overtaking or queuing dynamics depending on lane configurations. The interaction between the PDE and the ODE components occurs through flux constraints, which induce non-classical shock waves in the traffic density. We explore three classes of control strategies: centralized, decentralized, and quasi-centralized, which regulate the AVs' desired speeds to stabilize traffic and optimize selected cost functions. Global optimization, Model Predictive Control and reinforcement learning formulations are investigated. We demonstrate—both analytically and through simulations—how these control strategies can lead to improved traffic performance. We conclude by discussing the MegaVanderTest, a large-scale experiment involving 100 connected and automated vehicles, which provides empirical support for the modeling and control.

Limit cycle and asymptotic gait for a dynamic model of rectilinear locomotion

Paolo GIDONI, Università degli Studi di Udine, Italy

Biological and bio-inspired locomotion is usually described by recognizing periodic patterns, or gaits, in the movement of limbs or other body parts. But is the evolution of the system actually periodic? Or more properly, relative-periodic, since, presumably, each cycle will propel the animal (or robot) a little bit forward? The answer is often no, due, for instance, to inertia or elasticity. However, we might expect the behaviour to converge asymptotically to a relative-periodic one. In this talk we will introduce this issue considering, as a case study, a dynamic model of rectilinear crawling locomotion. We study the existence of a global periodic attractor for the reduced dynamics of the model, corresponding to an asymptotically relative-periodic motion of the crawler. The main result is of Massera-type, namely we show that the existence of a bounded solution implies the existence of the global periodic attractor for the reduced dynamics. Additional conditions and a counterexample for the existence of a bounded solution (and therefore of the attractor) will be briefly discussed. We conclude surveying the issue for some related models.

Nonlinear differential problems via variational, set-valued and topological methods

Roberto LIVREA, Università degli Studi di Palermo, Italy

The talk focuses on three different kinds of nonlinear differential problems with the aim of showing different methods for the study of ODEs.

In particular, referring to [2], a possible variational approach, based on [1], will be shown in order to assure infinitely many solutions for the following class of higher order ordinary differential equations

$$\begin{cases} -u^{(vi)} + Au^{(iv)} - Bu'' + Cu = \lambda f(x, u), & x \in [0, 1] \\ u(0) = u(1) = u''(0) = u''(1) = u^{(iv)}(0) = u^{(iv)}(1) = 0, \end{cases}$$

provided f is a continuous function satisfying a suitable oscillation behavior, as well as A , B and C are given real constants, while λ is a positive parameter belonging to a well determined interval.

Moreover, the results proved in [5] will be outlined, so that, by means of critical point theory for non-differentiable functionals (see [3]), the following periodic boundary value problem with the Sturm-Liouville equation having highly discontinuous nonlinearities

$$\begin{cases} -(pu')' + qu = \lambda f(x, u), & x \in [0, T], \\ u(0) = u(T), \quad u'(0) = u'(T), \end{cases}$$

will be investigated. Here, $p, q \in L^\infty([0, T])$ satisfy $p(0) = p(T)$, $q_0 = \operatorname{ess\,inf}_{[0, T]} q > 0$, $p_0 = \operatorname{ess\,inf}_{[0, T]} p > 0$, $\lambda > 0$ and $f : [0, T] \times \mathbf{R} \rightarrow \mathbf{R}$ is an almost everywhere continuous function.

Finally, the existence of at least one positive classical solution of the following two point nonlinear Dirichlet boundary value problem

$$\begin{cases} -u'' = f(x, u, u'), & x \in [a, b], \\ u(a) = u(b) = 0, \end{cases}$$

where $f : [a, b] \times \mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ is a continuous function, will be discussed. As detailed in [4], combining difference methods with Brouwer fixed point and Ascoli-Arzelà theorems, some well-known existence results will be re-proposed, but, as a novelty, the approximation of the solution, by using the solutions of the corresponding sequence of difference equations, will be illustrated.

References

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- [2] G. Bonanno and R. Livrea, A sequence of positive solutions for sixth-order ordinary nonlinear differential problems, *Electron. J. Qual. Theory Differ. Equ.* (2021), article 20
- [3] G. Bonanno and S.A. Marano, On the structure of the critical set of non-differentiable functions with a weak compactness condition, *Appl. Anal.* 89 (2010), 1–10
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- [5] R. Livrea and B. Vassallo, Three weak solutions to a periodic boundary Sturm-Liouville problem with discontinuous reaction, *Discrete Contin. Dyn. Syst. Ser. S* 18 (2025), 1660–1672

Mean-field limit of particle systems over hypergraphs

Nastassia POURADIER DUTEIL, INRIA Paris, France

We present a generalization of non-exchangeable particle systems with higher-order interactions, by removing two important assumptions that are usually made in particle systems: that of exchangeability of particles, and that of superposition of binary interactions. In the general model that we consider, individuals instead interact by groups, so that a full group jointly generates a non-linear force on any individual. This interaction is modeled by an underlying hypergraph. We derive the mean-field limit of the particle system (i.e. its limit as the number of particles tends to infinity), and show that it is determined by a Vlasov-type equation, where the limit of the hypergraph is given by a so-called unbounded-rank hypergraphon, and the mean-field force admits infinitely-many orders of interactions.

Derivation of cross-diffusion models in population dynamics: dichotomy, time-scales, and fast-reaction

Cinzia SORESINA, Università degli Studi di Trento, Italy

In population dynamics, cross-diffusion describes the influence of one species on the diffusion of another. A benchmark problem is the cross-diffusion SKT model, proposed in the context of competing species to account for stable inhomogeneous steady states exhibiting spatial segregation. Even though the reaction part does not present the activator-inhibitor structure, the cross-diffusion terms are the key ingredient for the appearance of spatial patterns [2]. From the modelling perspective, cross-diffusion terms naturally appear in the fast-reaction limit of a “microscopic” model (in terms of time scales) presenting only standard diffusion and fast-reaction terms, thus incorporating processes occurring on different time scales [5]. This talk presents recent applications of this approach, e.g., predator-prey [1,3] and mutualistic interactions, plant dynamics with autotoxicity effects [4], epidemiology, and metal surface corrosion.

References

- [1] M. Bisi, A. Bondesan, M. Groppi and C. Soresina, A kinetic model for prey-predator dynamics, (2025), in preparation
- [2] M. Breden, C. Kuehn and C. Soresina, On the influence of cross-diffusion in pattern formation, *J. Comput. Dyn.* 8 (2021), 213–240
- [3] L. Desvillettes and C. Soresina, Non-triangular cross-diffusion systems with predator-prey reaction terms, *Ric. Mat.* 68 (2019), 295–314
- [4] F. Giannino, A. Iuorio and C. Soresina, The effect of auto-toxicity in plant-growth dynamics: a cross-diffusion model, (2025), in preparation
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Travelling waves in tubes model of gravitational fingering

Sergey TIKHOMIROV, PUC-Rio, Brasil

We discuss gravitational fingering phenomenon - the unstable displacement of miscible liquids in porous media with the speed determined by Darcy's law. Such model is called incompressible porous medium equation (IPM). A similar phenomenon of viscous fingers plays an important role in petroleum engineering in case of highly viscous oil or certain enhanced oil recovery methods [1, 2]. Laboratory and numerical experiments show the linear growth of the mixing zone, and we are interested in determining the exact speed of propagation of fingers. Knowledge of the precise value of the speed of the fingers would allow to optimize injection scheme of certain enhanced oil recovery methods (for instance polymer and surfactant-polymer flooding [2]). The existing theoretical upper bounds for the growth rate of the mixing zone are higher than the observed speed from the numerical simulations [1]. We believe that one of the possible mechanisms of slowing down the fingers' growth is due to convection in the transversal direction [3].

To demonstrate effect of the convection in the transversal direction we introducing a semi-discrete model. The model consists of a system of advection-reaction-diffusion equations on concentration, velocity and pressure in several vertical tubes (real lines) and interflow between them. In the simplest setting of two tubes we show the structure of gravitational fingers - the profile of propagation is characterized by two consecutive travelling waves which we call a terrace. We prove the existence of such a propagating terrace for the parameters corresponding to small distances between the tubes [4]. While for multiple tubes the solution has more complicated structure than propagating terrace, structures similar to two-tubes model describe significant part of the solution. An important tool is introduction of so-called Transverse Flow Equilibrium (TFE) model, derived under realistic assumption that pressure gradient is mostly vertical. The TFE model is easier to simulate and in certain cases admits an exact solution. We establish rigorous relation between IPM and TFE models. Relation between travelling waves of IPM and TFE model is described via singularly perturbed system.

The talk is based on a joint talk with Yu. Petrova and Ya. Efendiev.

References

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- [2] F. Bakharev, A. Enin, K. Kalinin, Yu. Petrova, N. Rastegaev and S. Tikhomirov, Optimal polymer slugs injection profiles, *J. Comput. Appl. Math.* 425 (2023), article 115042
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Multiplicity results for boundary value problems associated with Hamiltonian systems

Wahid ULLAH, Università degli Studi di Trieste, Italy

In this talk, I will focus on the periodic and Neumann-type boundary value problems associated with Hamiltonian systems. I classify my talk into three parts. In the first part, I will discuss some extensions of the higher dimension Poincaré–Birkhoff theorem for coupled Hamiltonian systems. The systems we are coupling have completely different behaviours: the first one is a system with periodic Hamiltonian in the space variable, while the second one is either a system having generalized lower/upper solutions or a positively- (p, q) -homogeneous Hamiltonian system. In the second part, I will discuss some multiplicity results for Neumann-type boundary value problems. The final part of my talk is dedicated to our recent result concerning the multiplicity of solutions for Hamiltonian systems associated with mixed periodic and Neumann-type boundary conditions.

This talk is mostly based on a joint work with Professor Alessandro Fonda, resulting in the following papers.

References

- [1] A. Fonda and W. Ullah, Periodic solutions of Hamiltonian systems coupling twist with generalized lower/upper solutions, *J. Differential Equations* 379 (2024), 148–174
- [2] A. Fonda and W. Ullah, Periodic solutions of Hamiltonian systems coupling twist with an isochronous center, *Differ. Integral Equ.* 37 (2024), 323–336
- [3] A. Fonda and W. Ullah, Boundary value problems associated with Hamiltonian systems coupled with positively- (p, q) -homogeneous systems, *NoDEA Nonlinear Differ. Equ. Appl.* 31 (2024), article 41
- [4] W. Ullah, A multiplicity result for Hamiltonian systems with mixed periodic-type and Neumann-type boundary conditions, (2024), preprint

Posters

Controllability: a multivalued approach

Giulia DURICCHI, Università degli Studi di Modena e Reggio Emilia, Italy

Oscillation theory of half-linear difference equations

Ludmila LINHARTOVÁ, Masarykova Univerzita Brno, Czech Republic

Some recent extensions of the Poincaré-Birkhoff Theorem

Natnael Gezahegn MAMO, Università degli Studi di Trieste, Italy

Long-term dynamics of Duffing-type equations with applications to suspension bridges

Emanuele PASTORINO, Politecnico di Milano, Italy